

Einstein's hole dilemma and "gauge" freedom.

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Abstract

The issue of the physical equivalence between the different coordinate system in Einstein theory is revised. "Gauge" fixing influences results of measurements and physics are different in two different coordinate system. Spacetime metric generated by static spherically symmetric distribution of matter can be matched with wide family of vacuum solution and the exterior spacetime geometry could not be deduced directly from the interior perfect fluid solution, without reference to a "gauge" fixing or viceversa. The property of solutions in general relativity is indeed an observer dependent concept.

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I. INTRODUCTION

In the usual description in the framework of general relativity our spacetime have a pseudo-riemannian geometry. However, this an arbitrary chose and the general theory of relativity may be formulated in the language of Weyl geometry [1], for example.

General relativity describe the universe as grounded on differentiable arbitrary manifold M^4 enveloped by a principal bundle formed of isometric representations of a finite continuous Poincaré group. Einstein's principle of general relativity asserts the invariance under general coordinate transformations of the actions integral grounded on a M^4 manifold parameterized by variables $x^\mu, \mu = 0, 1, 2, 3$.

According to the theory of manifolds one can always introduce the coordinate chart through a map from an open set in the manifold M^4 to an open set of R^4 . This approach adopt an interpretation of the variables x^μ as mere mathematical parameters, devoid of any geometrical significance. The parameters x^μ then do not classify an operationally well-defined position in space and time, although they can be regarded as defining a chart on an abstract manifold. Such a manifold should not, however, be confused with the space of all events, which requires the presence of physical fields for its very definition [2].

There is also Einsteins famous hole argument in general relativity which asserts that the notion of a space-time point (in a manifold) has no physical meaning in a theory that is invariant under the group of space-time diffeomorphisms [9]. Evidently, the use of coordinates is optional, and that one could adopt a coordinate free description of same manifold M^4 , in this approach, the manifold points cannot correspond to operationally well-defined events. Thus in general relativity coordinates in manifolds are physically meaningless before specifying the metric tensor though they designate a particular point of the underlying mathematical manifold.

II. VARIATIONAL PRINCIPLE

As is well known field equation and conservation law of the relativity theory can be obtained from principle of least action. The same principle is the basis of the general relativity

$$S = \int \sqrt{-g} R(g^{\mu\nu}, \partial_\lambda g^{\mu\nu}) d^4x + \int \sqrt{-g} L^M(g^{\mu\nu}, \partial_\lambda g^{\mu\nu}, \Psi, \partial_\lambda \Psi) d^4x, \quad (1)$$

where respectively we have the action integral of the geometry and the action integral of matter; R is the Ricci scalar a function of $g^{\mu\nu}$ and their partial derivatives, L^M the Lagrangian density of the matter as a functional of the metric tensor and a set of non-gravitational fields Ψ .

Note that in abstract manifold the Ricci scalar and tensor $g_{\mu\nu}$ lose their geometrical meaning they had in a spacetime and now can be viewed only as a source for the metric. An unsatisfactory feature of general relativity is that the components of Lagrangian do not have any direct physical interpretation. Moreover, Albert Einstein in 1923 assumed a priori that both a metric and connection must be chosen, from the beginning, as dynamical variables. Note that an affine connection is not uniquely defined by the Lagrangian structure and can be at most an independent postulat of theory [10]. In this case the point dependent property of manifolds is linked with the fact that the units for measure of underlying geometry will be running units. For example, it could be a theory based on Einstein Hilbert action but endowed with space time of Weyl integrable structure [11].

The general relativity appears as a theory in which the gravity is described simultaneously by two fields the metric tensor and the matter fields, the latter being an essential part of the geometrical property of spacetime (emerged after solution of the field equations) manifesting its presence in almost all geometrical phenomena, such as curvature, geodesic motion and so on.

While the gravitational interaction are described by a doublet constituting of a metric tensor and a matter fields, the important aspects of general relativity is connected with conformal symmetry. The matter terms of the Lagrangian density contain the connection and hence a part of dynamical description of gravity, that is invariant in form under the conformal "rescaling" of the metric. This conformal symmetry is sufficient to guarantee the invariance of the Lagrangian under arbitrary changes of variables. It is then evident that the total Lagrangian may contain the terms for one or more physical fields, so that we can shift it by changes of variables from geometric action integral S^G to matter action integral S^M .

Note that the limit case is a flat space, in this frame, similarly as in electrodynamic the $g^{\mu\nu}$ are 10 gravitational potentials and directly interacts with matter. The Cristoffel symbols are the gravitational field strength. Since the gravitational potentials are not observable quantities it has no direct physical interpretation in general.

It was first recognized by Weyl [3] and fully developed by Cartan [4] - [7] that in such a systems one can always make a field redefinition (a change of variables) to another arithmetization of manifold via conformal mapping

$$g_{\mu\nu}(x) \rightarrow e^{2\alpha(x)} \tilde{g}_{\mu\nu}(x). \quad (2)$$

where $\alpha(x)$ is a differentiable real function of manifold parameters.

The tensor calculus on these manifolds is enriched by new properties, which are completely explained by the Weyl transformations of a few basic quantities. Taking as fundamental-tensor variation the finite transformation (2), we obtain by the Weyl transformations

$$\Gamma_{\mu\nu}^{\lambda} \rightarrow \tilde{\Gamma}_{\mu\nu}^{\lambda} = \Gamma_{\mu\nu}^{\lambda} + \delta_{\nu}^{\lambda} \partial_{\mu} \alpha + \delta_{\mu}^{\lambda} \partial_{\nu} \alpha - g_{\mu\nu} \partial^{\lambda} \alpha; \quad (3)$$

$$R_{\mu\nu} \rightarrow \tilde{R}_{\mu\nu} = R_{\mu\nu} - 2[g_{\mu\nu} g^{\rho\sigma} \partial_{\rho} \alpha \partial_{\sigma} \alpha - \partial_{\mu} \alpha \partial_{\nu} \alpha + \nabla_{\mu} \partial_{\nu} \alpha] - g_{\mu\nu} \nabla_{\lambda} \nabla^{\lambda} \alpha; \quad (4)$$

$$R \rightarrow \tilde{R} = e^{-2\alpha} [R - 6g^{\rho\sigma} \partial_{\rho} \alpha \partial_{\sigma} \alpha - 6\nabla_{\lambda} \nabla^{\lambda} \alpha]; \quad (5)$$

where δ_{μ}^{ν} is the Kronecker delta function and ∇_{μ} is respectively the covariant differential operator constructed out of $g_{\mu\nu}(x)$. Eqs.(3)–(5) describe the structural changes of the basic tensors of the differential calculus.

This change of variables lead us to extension of Riemann connection by Weyl transformation implies the extension of the Poincaré group, to the conformal group. This is possible provided that the Weyl transformation act on any field representation Ψ according to the low

$$\Psi(x) \rightarrow \tilde{\Psi}(x) = e^{w_{\Psi} \alpha(x)} \Psi(x),$$

.

It is important to note that the two geometrical structures, the metric and arithmetization, are fundamentally independent geometrical objects. Thus the theory can be expressed in terms of infinite number of related charts.

This approach consists of introducing an extra geometrical entity in a manifold a 1-form field, for example, in terms of which the Riemannian compatibility condition between the metric g and the connection Γ is redefined. Then a group of transformations which evolves both g and "matter field", is defined by requiring that under these change of variables the new compatibility condition remain invariant. In a certain sense, this new invariance group

include the conformal transformation as subgroup. In other terms this "ordinary matter" may be represented in disguise in infinite number of way as a gravity component or as a conformally invariant matter fields. Once matter has been coupled to gravity in a frame one has a freedom to make a change of variables to any other frame. So property of Einstein Hilbert action in general relativity is indeed an observer dependent concept.

III. FIELD EQUATIONS

We can derive the field equation from the variational equation

$$\frac{\delta S}{\delta g^{\mu\nu}(x)} \equiv \frac{\delta(S^G + S^M)}{\delta g^{\mu\nu}(x)} = 0,$$

stating the invariance of the total action under change of variables. Since we have

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}^M, \quad (6)$$

this equation is usually understood as the equation which relates spacetime geometry to the distribution and motion of matter field. Starting from the Hilbert - Einstein equation, we must bear in mind that inherent in this equation is the coordinates x represent only a certain manifold, fixed by selectable arithmetization. In addition there is the Bianchi identity a simple consequence of these symmetry properties is that the field equations alone are not enough to determine a gravitational system, while these equations are a set of 6 nonlinear partial differential equations for the 10 metric components. Einsteins equations determine the solution of a given physical problem up to four arbitrary functions.

One of the largest concentrations of literature within the area of relativistic gravity theories is interpretations of exact solutions of field equations. Some of them are discovered at the early stage of development of relativistic theories, but up to now they are often considered as equivalent representations of some "unique" solution.

Evidently, a structure of space-times is mathematically represented by Einsteins equations (6) and four co-ordinate conditions [12], which considered independent of the action

$$C(x)g_{\mu\nu} = 0, \quad (7)$$

where $C(x)$ - some algebraic or differential operators. Thereby for any four of components $g_{\mu\nu}$ emerge the relations with remaining six and, probably, any others, known functions.

Certainly, equations (7) cannot be covariant for the arbitrary transformations of independent variables, and similarly should not contradict Einsteins equations or to be their consequence. Moreover, these four equations will not be transformed according to any rules, but simply replaced by hand with the new. From geometrical point of view one has to introduce an additional mathematical structure - describing some specific principle of construction of space-time model is responsible for measuring the distances - the "gauge". In general relativity "gauge" and coordinate transformation means the same thing. On the other words "gauge" is a rule for reception of "coordinate system" on a single manifold M (e.g. harmonic, isotropic, curvature coordinates). This "gauge" is the unphysical degree of freedom and we must fix the "gauge" or extract some invariant quantities to obtain physical results [13].

The unknown components of metric tensor $g_{\mu\nu}$ are determined from the solutions of Einstein's field equations. Consequently, the geometrically interpreted co-ordinate system of obtained space-time and any relationship it derives from equations (6), (7) emerge a posteriori [12]. Moreover, property of this co-ordinate system will depend from initial and boundary conditions for (6), (7). An intriguing consequences of the above discussion is the "gauge" freedom can be expected in relation with some connection to problems in quantum physics. Generally speaking, occurrence of the observer ("gauge" fixing) influences results of measurements and physics are different in two different "gauges" [8].

The distinct geometrical and physical picture of the same phenomena may arise in a different "coordinate systems". The physical content of this point of view can be stated in the following simple way: the property of a "matter" are not the same for the different "coordinate system" is chosen.

A. Equivalence frames

The important feature of the gravity theory is connected with the conformal symmetry. It is well known, since the pioneering paper of Jordan [14] that the action is invariant under local transformations of units that are under general conformal transformations, or sometimes called Weyl rescaling:

$$ds^2 \rightarrow d\tilde{s}^2 = e^{2\alpha(x)} ds^2. \quad (8)$$

where $\alpha(x)$ a local arbitrary function of x .

This method of conformal transformation provides a clear and powerful technique, free from mathematical ambiguity, but nevertheless requires careful consideration from the physical point of view.

Among all conformally related frames one distinguishes two frames: Jordan's and Einstein's. Note that, unless a clear statement of what is understood by "equivalence of frames" is made, the issue which is the physical conformal frame is a semantic one. For example, by shifting a mass terms in Lagrangian one can construct four related but inequivalent theories in Jordan and Einstein frame [19].

In the literature, the physicists do not agree with each other about the equivalence of the two frames (see review in [16]). However, the meaning of the equivalence between the Jordan frame and the Einstein frame is not assuming the additional equations (7). These equations put by hand and not covariant. This issue is critical for the interpretation of the predictions of a given theory of gravity since these seem to be deeply affected by the choice of the coordinate conditions [8]. For concreteness, let us consider "harmonic gauge" [17],

$$g_{\mu\nu}\Gamma_{,\mu\nu}^{\lambda} = 0. \quad (9)$$

which usually assumed as the analogue of Lorenz gauge, $\partial A = 0$, in electromagnetism. However this analogy is the most superficial: this or that gauge in nonrelativistic theory is a problem of exclusively convenience, it's this or that expedient does not influence in any way on a values of physical quantities and it is not related to observation requirements, - whereas the choice of co-ordinate system is related to all it essentially.

In fact, there are the related but inequivalent theories in Jordan and Einstein frame. The reason is very simple. If we use the same conformal transformations, like the (2), in both the equations (6) and (7), then the in and out states are not the same in the two frames. If one postulates that the field equations are invariant with respect to conformal transformations (7), one obtains in addition transformations of co-ordinate conditions

$$g_{\mu\nu}\Gamma_{,\mu\nu}^{\lambda} = \tilde{g}_{\mu\nu}\tilde{\Gamma}_{,\mu\nu}^{\lambda} + \partial_{\mu}\alpha. \quad (10)$$

As a result, since the Einstein field equations are undetermined; gravity theory cannot achieve the harmonic metric for any α functions but only when α is taken a constant. One

must assume that two frames represent not the same set of physical gravitational and non-gravitational fields. In fact, two conformally connecting spaces $V^4(g)$ and $\tilde{V}^4(g)$ are given not in the same manifold. Consequently, under this conformal transformation the solution of some initial physical problem will be transformed onto a solution of a completely different problem. Thus, applying the same coordinate conditions in different physical requirements, we arrive at dissimilar physical theories, because we are solving different equations.

On the other hand each scalar-tensor theory can be considered as general relativity plus conformally invariant scalar fields [18]. The gravitational interaction for scalar tensor theories is taken into account by the Einstein equations, which are generally written in the form (6). The left-hand-side of equation is constructed from the geometrical properties of the space-time, while $T_{\mu\nu}$ is the energy momentum tensor of matter fields. One can in principle assume gauge-dependence of right-hand-side of equation (6) as a variety of matter fields with different equations of state. Now, if we consider, the system (6), (7) as equations for same "gauge" fixing then the Jordan's and Einstein's conformal frames can be viewed as a different "matter source" of energy momentum tensors $T_{\mu\nu}$ of Einstein's equations.

It is evident that in different conformal frame representations are neither mathematically, nor physically equivalent.

IV. MATCHING OF SPACETIMES

There are the relationship between the "gauge freedom" of General Relativity and the hole argument. Actually standard Einstein hole argument can be written by reverting to spacetime model of perfect fluid configuration with vacuum background. An important aspect in General Relativity is the analysis of how to match two spacetimes. Obviously, there are infinite ways to identify the manifolds, all of them equally valid a priori. This freedom leads to the gauge dependence of the emerged spacetime and of any other geometrically defined tensors. In particular, the matched spacetime cannot be thought to exist beforehand. Another aspect is that the matching conditions involve exclusively tensors on the identified boundary – and hence any coordinate system in both spacetimes is equally valid. Most of the difficulties arise from the fact that the matching conditions are imposed in specific coordinate systems. The matching involves finding an identification of the boundary and that this should not be fixed a priori and fields to be matched are gauge dependent too.

From the mathematical point of view, the most simple and satisfactory expression for the matching conditions is, following Linchnerowicz, the assumption that there exists a system of co-ordinates in which the metric tensor satisfies the continuity conditions. Let ξ^i be a coordinate system on Σ where Σ is an abstract copy of any of the boundaries. Greek indices range over the coordinates of the 4-manifold and Roman indices over the coordinates of the 3-surfaces. Continuity conditions require a common coordinate system on Σ and this is easily done if one can set $\xi_+^i = \xi_-^i$.

Let (V^\pm, g^\pm) be four-dimensional spacetimes with non-null Σ^\pm . The junction/shell formalism constructs a new manifold \mathcal{M} by joining one of the distinct parts of V^+ to one of the distinct parts of V^- by the identification $\Sigma^+ = \Sigma^- \equiv \Sigma$. The matching conditions require the equality of the first and second fundamental forms on Σ^\pm . Tangent vectors to Σ^\pm are obtained by $e_i^{\pm\alpha} = \frac{\partial x^\alpha}{\partial \xi^i}$. There are also unique (up to orientation) unit normal vectors n_\pm^α to the boundaries. We choose them so that if n_+^α points towards V^+ then n_-^α points outside of V^- or viceversa. Clearly the sign of the normal vectors are crucial since e.g. n_-^α points away from the portion of V^- which will be used in forming \mathcal{M} . The three basis vectors tangent to Σ are

$$e_i^\alpha = \frac{\partial x^\alpha}{\partial \xi^i} \quad (11)$$

which give the induced metric (first fundamental form) on Σ by

$$q_{ij} = \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} g_{\alpha\beta}. \quad (12)$$

The extrinsic curvature (second fundamental form) is given by

$$\begin{aligned} K_{ij} &= \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} \nabla_\alpha n_\beta \\ &= -n_\gamma \left(\frac{\partial^2 x^\gamma}{\partial \xi^i \partial \xi^j} + \Gamma^\gamma_{\alpha\beta} \frac{\partial x^\alpha}{\partial \xi^i} \frac{\partial x^\beta}{\partial \xi^j} \right). \end{aligned} \quad (13)$$

Then matching conditions are simply

$$q_{ij}^+ = q_{ij}^-, \quad (14)$$

$$K_{ij}^+ = K_{ij}^-. \quad (15)$$

If both (14) and (15) are satisfied we refer to Σ as a boundary surface. If only (14) is satisfied then we refer to Σ as a thin-shell.

Consider be a bounded, closed spacetime region V^- on which the metric field g^- is the only one present, so that inside V^- , the metric g^- obeys the Einsteins field equations (6). Given a solution $g^-(x)$ everywhere inside of and on the boundary of V^- , including all the normal derivatives of the metric up to any finite order on that boundary, this data still does not determine a unique solution outside V^- , because an unlimited number of other solutions can be generated from it by those diffeomorphisms that are identity inside V^- , but differ from the identity outside V^- . The resulting metric $g^+(x)$ will agree with $g^-(x)$ inside of and on the boundary of V^- , but will differ from it outside V^- .

A. Matching of incompressible liquid sphere with vacuum background

As is well known, static solutions of Einsteins equations with spherical symmetry (the exterior and interior Schwarzschild solutions) are staples of courses in general relativity. In the following analysis we assume a perfect fluid incompressible liquid sphere as a simplest model for matter field.

Writing two static spherically symmetric spacetimes V^+ and V^- with signature $(-+++)$. one can suppose that the metrics $g_{\alpha\beta}^+(x_+^\gamma)$ and $g_{\alpha\beta}^-(x_-^\gamma)$ in the coordinate systems x_+^γ and x_-^γ are of the forms

$$ds^2 = -B^-(r, t)dt^2 + A^-d(r, t)dr^2 + R^-(r, t) (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (16)$$

and

$$ds^2 = -B^+(r, t)dt^2 + A^+d(r, t)dr^2 + R^+(r, t) (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (17)$$

where $A^\pm(r, t)$, $B^\pm(r, t)$ and $R^\pm(r, t)$ are of class C^2 .

Within these spacetimes define two non-null 3-surfaces Σ^+ and Σ^- with metrics $q_{ij}^+(\xi_+^k)$ and $q_{ij}^-(\xi_-^k)$ in the coordinates ξ_+^k and ξ_-^k which decompose each of the 4-spacetimes into two distinct parts. The parametric equation for Σ is of the form

$$r - r_b = 0. \quad (18)$$

The induced metric on the Σ by the two solutions (16) and (17) is

$$q_{ij}^\pm d\xi_\pm^i d\xi_\pm^j = -B^\pm(r, t)dt^2 + R^\pm(r, t) (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (19)$$

therefore we must have on the Σ according to equality of the first fundamental form

$$B^-(r_b, t) = B^+(r_b, t), C^-(r_b, t) = R^+(r_b, t) \quad (20)$$

We can impose that the Σ be also characterized by equality of the second fundamental form too. This condition leads to

$$K_{11} = \frac{1}{2} \frac{B'(r_b, t)}{\sqrt{A(r_b, t)}}, K_{22} = \frac{1}{2} \frac{R'(r_b, t)}{\sqrt{A(r_b, t)}} \quad (21)$$

where prime denote derivation with respect to r . Continuity of the second fundamental form is merely equivalent to

$$B^-(r_b, t) = B^+(r_b, t), R^-(r_b, t) = R^+(r_b, t), \\ R^{+'}(r_b, t)^2 A^- = R^{-'}(r_b, t)^2 A^+, B^{+'}(r_b, t)^2 A^- = B^{-'}(r_b, t)^2 A^+ \quad (22)$$

Note that in the case of incompressible liquid model with curvature coordinates the requirement of matching spacetimes embraces continuity of the metric function. However, the derivative of g_{11} is inescapably discontinuous, and the derivative of g_{00} is already continuous without the necessity of requiring it [20]. This is of course the same that happens when trying to match other exterior and interior spherically symmetric solutions. The solution of Einstein equation for (16) can be reduced to

$$ds^2 = -\frac{(1 + r^2/S^2)^2}{1 + r^2/r_b^2} dt^2 + \frac{dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)}{(1 + r^2/r_b^2)^2}. \quad (23)$$

A brief computation yields

$$\rho = \frac{12}{r_b^2}; \quad p = \frac{4}{S^2 r_b^4} \frac{r_b^2(r_b^2 - 2S^2) - (2r_b^2 - S^2)r^2}{1 + r^2/S^2}. \quad (24)$$

where S arbitrary constant.

After finding interior solutions, we can then connect them to the exterior vacuum solutions. We take r_b to be the point where $p(r) = 0$, and use the values of $A^-(r_b), B^-(r_b), C^-(r_b), A^{-'}(r_b), B^{-'}(r_b), C^{-'}(r_b)$ from the solutions as conditions to determine the unknown integration coefficients from the vacuum case. As we pointed out unlimited number of spherically symmetric vacuum solutions can be obtained outside V^- . It is easy to show, however, that we can match the interior solution with the most general spherically symmetric vacuum solution [21]

$$ds^2 = \frac{\rho^{+'}(r)^2}{4\rho^+(r)} \left(1 - \frac{2\mu}{\sqrt{\rho^+(r)}}\right)^{-1} dt^2 + \left(1 - \frac{2\mu}{\sqrt{\rho^+(r)}}\right) dr^2 + \rho^+(r) (d\theta^2 + \sin^2 \theta d\phi^2). \quad (25)$$

where $\rho^+(r)$ is a arbitrary function of r . To perform the matching, we impose the arbitrary function $\rho^+(r)$ in a polynomial form. The conditions (14) must be fulfilled at the boundary where $p = 0$. These continuity conditions provide us with the information needed to find values for the arbitrary polynomial coefficients for the vacuum solutions. After matching the solutions in this manner, $B'(r)$ is not necessarily continuous at the boundary.

Now in order to justify calling the geometry an exact solution we need an explicit definition for the constant in these solutions. The integration constants of solution (23) and (25) are arbitrary. Obviously, it is possible to match the solution (23) to the vacuum (25) metric with unlimited number of arbitrary function $\rho^+(r)$.

Finally, we observed some surprising ambiguity of interpretation to a choice of function $\rho^+(r)$ generally corresponds to a choice of state within the vacuum. For example, the solution (25) with different choose of arbitrary function to $\rho^+(r)$ have the same spatial boundary behavior but have different property, and so they represent different vacuum states. Thus, from the point of view that the field equations (6) is just a formal device to arrive at the space time, information about the possible vacua of the theory, and the space of states in each vacuum, is not encoded directly in the (6), only indirectly through the "gauge" fixing and boundary conditions required of the equations.

It is seen by inspection that all junction conditions (22) are satisfied. We can therefore say that spacetime metric generated by static spherically symmetric distribution of perfect fluid incompressible matter can be matched with wide family of vacuum solution by suitable choose the integration constants or vice versa. We therefore can to see what any explicit constraints on the exterior spacetime geometry could not be deduced directly from the interior perfect fluid solution, without reference to a "gauge" fixing or viceversa.

V. CONCLUSION

In this article, we clarify the notion of arithmetization, "gauges" and coordinate transformations in relativistic theories, which is necessary to understanding the physical equivalence between the different coordinate system. Einstein Hilbert action is ambiguously decomposed into the sum of physical term, which represent the gravitational effects, and a pure geometrical term which represent the spurious gravitational effects associated with many-fold arithmetization. The matter terms of the Lagrangian density contain the connection

and hence a part of dynamical description of gravity, that is invariant in form under the conformal "rescaling" of the metric. In other words the matter terms may be represented in disguise in infinite number of way as a gravity component or as a conformally invariant fields. Once matter has been coupled to gravity in a frame one has a freedom to make a change of variables to any other frame. So property of Einstein Hilbert action in general relativity is indeed an observer dependent concept.

In relativistic theory, we must always write in addition to fields equation (6) four coordinate conditions (7). These "gauges" may describe different physical solutions of Einstein equations with the same space arithmetization. We have shown the "gauge" fixing influences results of measurements and physics are different in two different coordinate system.

The gravitational interaction are described by a doublet constituting of a metric tensor and a matter fields, the important feature of general relativity is connected with conformal symmetry. In particular, according to this view, general relativity may be rewritten in terms an arbitrary conventional geometry [24] and the geometry of space-time can be freely chosen by the theoretician [23]. In method of conformal transformation, we always treat two spacetimes. First is the space-time for one frame and the other is the space-time for another frame. Note that the two space-times for these frames are distinct. The conformal transformations are not diffeomorphisms of the single manifold M , and the transformed metric $\tilde{g}_{\mu\nu}$ is not simply the metric $g_{\mu\nu}$ written in a different coordinate system these metrics describe different gravitational fields and different physics.

Eq. (8) is a rather curious equation because it not covariant for the arbitrary transformations of independent variables. In this case the metric is left unchanged, although its coordinate representation varies. In short, Eq. (8) gives a relation between variables on two different space-times.

Evidently, a structure of space-times can be mathematically represented with cosmological and coupling *constants*; the conformally changed Einstein equations have the advantage of non-vanishing modified terms together with *dynamical* cosmological and gravitational coupling terms.

The Einstein's hole argument states a spacetime and a gravitational field form an indivisible unit: no field, no spacetime. We consider the vacuum static spherically symmetric solutions of general relativity to illustrate this. One might represent the metric tensor components and it's first derivatives on a boundary hypersurface (23) with some constrains from

the field equations, would uniquely determine the solution in neighborhood spacetime. But no such boundary condition can do this: any solution (25) can be transformed to other by a suitable choice of arbitrary function $\rho(r)$. The field equations cannot even uniquely determine the geometry of a spacetime on which a solution is defined.

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